NDEJJE SENIOR SECONDARY SCHOOL

Uganda Advanced Certificate of Education

MARKING GUIDE FOR MOCK SET 4 EXAMINATIONS 2017

APPLIED MATHEMATICS

Paper 2

SNo.	Working	Marks
1	(i). $P(A/B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(B).P(A/B) = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$ $P(A \cup B)' = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$ $= 1 - \left[\frac{8}{15} + \frac{1}{3} - \frac{1}{15}\right] = 1 - \frac{4}{5} = \frac{1}{5} = 0.2$ (ii). $P(B'/A) = \frac{P(A \cap B')}{P(A)} = \frac{P(A) - P(A \cap B)}{P(A)}$ $= \left(\frac{8}{15} - \frac{1}{15}\right) \div \frac{8}{15} = \frac{7}{15} \times \frac{15}{8} = \frac{7}{8} = 0.875$	 **B1** **M1 A1** **M1 A1**
2	$\underset{\sim}{\boldsymbol{v}} = t\begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} + t^2\begin{pmatrix} 0 \\ 2 \\ -\sin 2t \end{pmatrix} = \begin{pmatrix} 6t \\ 4t + 2t^2 \\ -t^2 \sin 2t \end{pmatrix}$ $\underset{\sim}{\boldsymbol{a}} = \frac{dv}{dt} = \begin{pmatrix} 6 \\ 4 + 4t \\ -2t^2 \cos 2t - 2t \sin 2t \end{pmatrix}$ $\underset{\sim}{\boldsymbol{F}} = m\underset{\sim}{\boldsymbol{a}} = 3\begin{pmatrix} 6 \\ 4 + 4t \\ -2t^2 \cos 2t - 2t \sin 2t \end{pmatrix}$ $= \begin{pmatrix} 18 \\ 12 + 12t \\ -6t^2 \cos 2t - 6t \sin 2t \end{pmatrix}$ The rate of doing work at any time, t, is given by; $P(t) = \underset{\sim}{\boldsymbol{F}}.\underset{\sim}{\boldsymbol{v}} = \begin{pmatrix} 18 \\ 12 + 12t \\ -6t^2 \cos 2t - 6t \sin 2t \end{pmatrix}.\begin{pmatrix} 6t \\ 4t + 2t^2 \\ -t^2 \sin 2t \end{pmatrix}$ The rate of doing work after two seconds is given by;	 **B1** **B1** **M1**

	$$P(2)=\begin{pmatrix}18\\12+24\\-24\cos 4-12\sin 4\end{pmatrix}\cdot\begin{pmatrix}12\\8+8\\-4\sin 4\end{pmatrix}$$ $$=\begin{pmatrix}18\\36\\24.769\end{pmatrix}\cdot\begin{pmatrix}12\\16\\-3.027\end{pmatrix}$$ $$P(2)=216+576-74.976=717.024\text{ W}$$	**M1** **A1**
3	working value, $v=\dfrac{22.5}{0.426}=52.816901$ maximum value, $$v_{max}=\frac{22.5+0.05}{0.426-0.0005}=\frac{22.55}{0.4255}=52.996475$$ maximum error, $\Delta v=v_{max}-v$ $=52.996475-52.816901=0.179574$	**B1** **M1 B1** **M1 A1**
4	<table><tr><th>x</th><th>$F(x)$</th><th>$f(x)$</th><th>$xf(x)$</th></tr><tr><td>1</td><td>λ</td><td>λ</td><td>λ</td></tr><tr><td>2</td><td>4λ</td><td>3λ</td><td>6λ</td></tr><tr><td>3</td><td>9λ</td><td>5λ</td><td>15λ</td></tr><tr><td>Total</td><td></td><td>9λ</td><td>22λ</td></tr></table> (a). $$\sum_{all\ x} f(x)=1,\quad \Rightarrow 9\lambda=1,\quad \Rightarrow \lambda=\frac{1}{9}$$ (b). $$E(X)=\sum_{all\ x} xf(x)=22\lambda=22\times\frac{1}{9}=\frac{22}{9}\approx 2.444$$	**B1** –for f(x) **M1 A1** **M1 A1**
5	(a). $$m=150\text{ g}=0.15\text{ kg}, s=10\text{ cm}=0.1\text{ m}, v=0\text{ m s}^{-1}$$ $$u=215\text{ km h}^{-1}=\frac{215\times 1000}{3600}=59.722\text{ m s}^{-1}$$ $$v^2=u^2+2as$$ $$0^2=59.722^2+2a\times 0.1$$ $$a=-\frac{59.722^2}{0.2}=-17833.586\text{ m s}^{-2}$$ Let F_r be the resistance due to the block. Using Newton's second law of motion, $0-F_r=0.15a$, $$\Rightarrow F_r=-0.15\times(-17833.586)=2675.038\text{ N}$$ (b). $$v^2=u^2+2as=59.722^2+2\times(-17833.586)\times 0.04$$ $$=2140.03$$ $$v=\sqrt{2140.03}=46.26\text{ m s}^{-1}$$	**B1** **M1 A1** **M1** **A1**

6

(i).

$18'$	$24'$	$30'$
y	0.4748	0.4770

B1 (mobile)

$$\frac{y-0.4748}{0.4770-0.4748}=\frac{18-24}{30-24}$$

M1

$$\frac{y-0.4748}{0.0022}=-1$$
$$y=-0.0022+0.4748=0.4726$$
$$\therefore \tan 25°18'=0.4726$$

A1

(ii).

$30'$	x	$36'$
0.4770	0.4775	0.4791

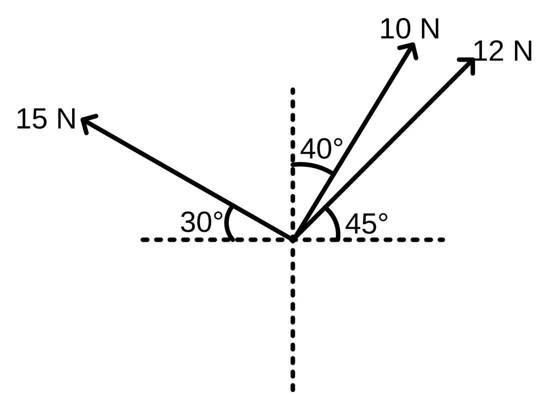
$$\frac{x-30}{36-30}=\frac{0.4775-0.4770}{0.4791-0.4770}$$

M1

$$\frac{x-30}{6}=0.2381$$
$$x=0.2381\times 6+30=31.4286$$
$$\therefore \tan^{-1}(0.4775)=25°31.4286'$$

A1

7



$$\underset{\sim}{\boldsymbol{F}}=\begin{pmatrix}10\sin 40°\\10\cos 40°\end{pmatrix}+\begin{pmatrix}-15\cos 30°\\15\sin 30°\end{pmatrix}+\begin{pmatrix}12\cos 45°\\12\sin 45°\end{pmatrix}=\begin{pmatrix}1.92278\\23.64573\end{pmatrix}$$

B1 M1

$$\left|\underset{\sim}{\boldsymbol{F}}\right|=\sqrt{1.92278^2+23.64573^2}=23.72\text{ N}$$

A1

Let the resultant be in the direction $E\ \theta\ N$.

$$\theta=\tan^{-1}\left(\frac{23.64573}{1.92278}\right)=85.35°,\qquad \Longrightarrow \text{Direction}=E\ 85.35°\ N$$

M1 A1

8

X	Y	R_X	R_Y	$d=R_X-R_Y$	d^2
A	C_3	1	5.5	-4.5	20.25

O	D_2	7	3	4	16
B	D_1	2.5	1	1.5	2.25
F	P_8	8	7.5	0.5	0.25
E	P_8	6	7.5	-1.5	2.25
C	D_2	4	3	1	1
D	C_3	5	5.5	-0.5	0.25
B	D_2	2.5	3	-0.5	0.25
					$\sum d^2$ =42.5

B1 –both rankings
B1 -$\sum d^2$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 42.5}{8(8^2-1)} = 0.4940$$

M1 A1

Comment: Insignificant at 1%.

B1

9 (a).

$$y_n = \sin^2 x_n\,, \qquad h = \frac{3-1}{5} = \frac{2}{5} = 0.4$$

n	x_n	y_0, y_5	$y_1, \dots, y_4$
0	1	0.7081	
1	1.4		0.9711
2	1.8		0.9484
3	2.2		0.6537
4	2.6		0.2657
5	3	0.0199	
Sums		0.728	2.8389

B1 –for h
B1 –for x_n
B1 – for y_0, y_5
B1 –for $y_1, \dots, y_4$
B1 – for sum $y_0 + y_5$
B1 –for sum y_1 to y_4

$$\int_1^3 \sin^2 x\,dx \approx \frac{1}{2}h\{(y_0 + y_5) + 2(y_1 + \dots + y_4)\}$$
$$\approx \frac{1}{2} \times 0.4 \times [0.728 + 2 \times 2.8389] = 1.281 \text{ (3 d. p)}$$

M1 A1

(b).

$$\text{Actual value } = \int_1^3 \sin^2 x\,dx = \frac{1}{2}\int_1^3 (1 - \cos 2x)\,dx$$
$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_1^3$$

M1

$$= \frac{1}{2}\left[\left(3 - \frac{\sin 6}{2}\right) - \left(1 - \frac{\sin 2}{2}\right)\right] = 1.297 \text{ (3 d. p)}$$

B1

$$\text{Relative error } = \left|\frac{\text{Error}}{\text{Actual value}}\right| = \left|\frac{1.297 - 1.281}{1.297}\right| = 0.0123$$

M1 A1

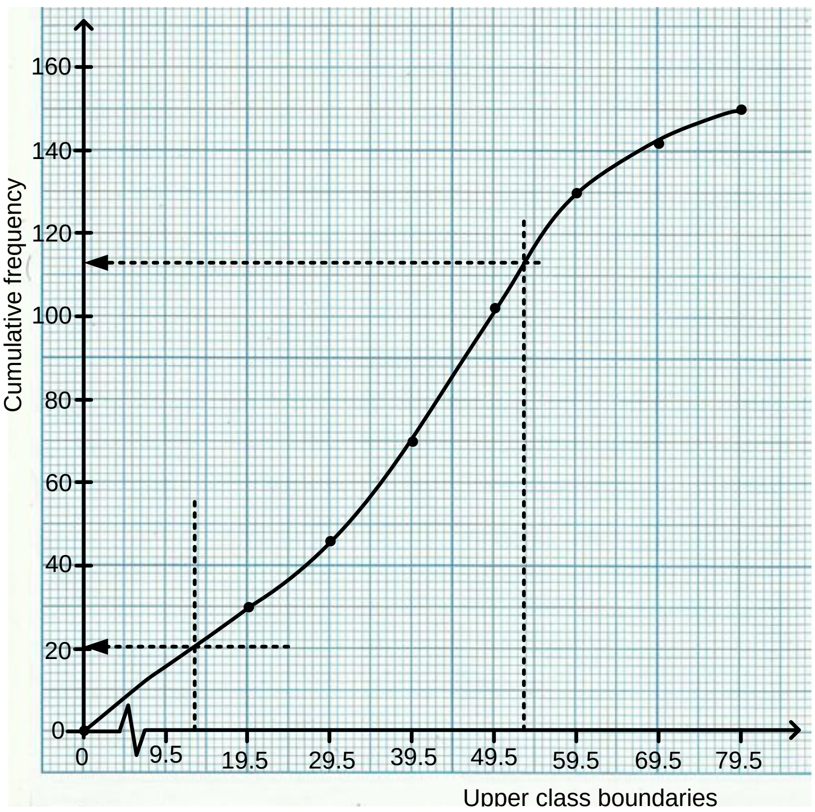
The relative error can be reduced by increasing the number of strips within the given interval.

10 (a).

Class	$C.F$	f	x	fx	c	f/c	Class boundaries

$0 \leq W \leq 19$	30	30	14.5	435	20	1.5	$-0.5 - 19.5$
$20 \leq W \leq 29$	46	16	24.5	392	10	1.6	$19.5 - 29.5$
$30 \leq W \leq 39$	70	24	34.5	828	10	2.4	$29.5 - 39.5$
$40 \leq W \leq 49$	102	32	44.5	1424	10	3.2	$39.5 - 49.5$
$50 \leq W \leq 59$	130	28	54.5	1526	10	2.8	$49.5 - 59.5$
$60 \leq W \leq 69$	142	12	64.5	774	10	1.2	$59.5 - 69.5$
$70 \leq W \leq 79$	150	8	74.5	596	10	0.8	$69.5 - 79.5$
Total		150		5975			

B1 –for f
B1 –for x
B1 –for fx
B1 –for f/c



B1 –axes
B1 –all points
B1 –smooth curve

The estimate number of patients who weigh between 24.5 kg and 54.5 kg $= 113 - 20 = 93$. **A1**

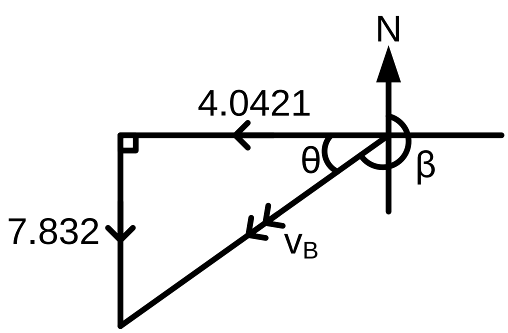
(b). (i).

Mean weight, $$\overline{x} = \frac{\sum fx}{\sum f} = \frac{5975}{150} = 39.833$$ **M1 A1**

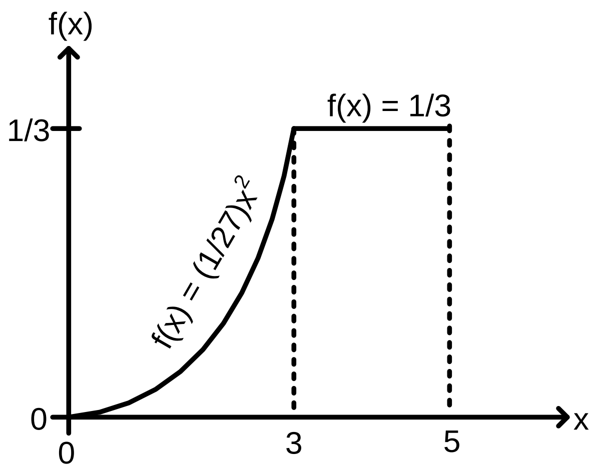
(ii).

$$L_m = 39.5, \Delta_1 = 3.2 - 2.4 = 0.8, \Delta_2 = 3.2 - 2.8 = 0.4, c = 49.5 - 39.5 = 10$$

	$\text{Modal weight} = L_m + \left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right)c = 39.5 + \left(\frac{0.8}{0.8+0.4}\right)\times 10$ $= 39.5 + 6.667 = 46.167$	**M1** **A1**
11	(a). ${}_A\underset{\sim}{\boldsymbol{r}}_B\,(t) = {}_A\underset{\sim}{\boldsymbol{r}}_B + t\,{}_A\underset{\sim}{\boldsymbol{v}}_B = \begin{pmatrix}2\\20\\18\end{pmatrix} + t\begin{pmatrix}-2\\1\\-5\end{pmatrix} = \begin{pmatrix}2-2t\\20+t\\18-5t\end{pmatrix}$ km For closest approach, ${}_A\underset{\sim}{\boldsymbol{r}}_B\,(t).\,{}_A\underset{\sim}{\boldsymbol{v}}_B = 0$ $\begin{pmatrix}2-2t\\20+t\\18-5t\end{pmatrix}.\begin{pmatrix}-2\\1\\-5\end{pmatrix} = 0$ $-4+4t+20+t-90+25t = 0$ $t = \frac{74}{30} = \frac{37}{15} \approx 2.467$ hours ${}_A\underset{\sim}{\boldsymbol{r}}_B\left(\frac{37}{15}\right) = \begin{pmatrix}2\\20\\18\end{pmatrix} + \frac{37}{15}\begin{pmatrix}-2\\1\\-5\end{pmatrix} = \frac{1}{15}\begin{pmatrix}-44\\337\\85\end{pmatrix}$ km $D_{min} = \left\|{}_A\underset{\sim}{\boldsymbol{r}}_B\left(\frac{37}{15}\right)\right\| = \frac{1}{15}\sqrt{(-44)^2+337^2+85^2} = \sqrt{122730}$ $= 350.328$ km	**B1** **M1 M1 –** dotting & equating to zero **A1** **M1** **M1** **A1**
	(b). $\underset{\sim}{\boldsymbol{v}}_G = \begin{pmatrix}6\sin 25^\circ\\-6\cos 25^\circ\end{pmatrix}$ m s^{-1}, $\quad {}_B\underset{\sim}{\boldsymbol{v}}_G = \begin{pmatrix}-7\cos 20^\circ\\-7\sin 20^\circ\end{pmatrix}$ m s^{-1} ${}_B\underset{\sim}{\boldsymbol{v}}_G = \underset{\sim}{\boldsymbol{v}}_B - \underset{\sim}{\boldsymbol{v}}_G$ $\underset{\sim}{\boldsymbol{v}}_B = {}_B\underset{\sim}{\boldsymbol{v}}_G + \underset{\sim}{\boldsymbol{v}}_G = \begin{pmatrix}-7\cos 20^\circ\\-7\sin 20^\circ\end{pmatrix} + \begin{pmatrix}6\sin 25^\circ\\-6\cos 25^\circ\end{pmatrix}$ $= \begin{pmatrix}-4.0421\\-7.8320\end{pmatrix}$ m s^{-1} $\left\|\underset{\sim}{\boldsymbol{v}}_B\right\| = \sqrt{(-4.0421)^2+(-7.8320)^2} = 8.8136$ m s^{-1}	**B1 B1** **B1**



	$\tan\theta = \frac{7.832}{4.0421}, \quad \Rightarrow \theta = 62.70^\circ$ $\beta = 270 - 62.70 = 207.30^\circ$ The true velocity of the bird is 8.8136 m s^{-1} on a bearing 207.30°.	**M1** **A1**

12	(a). $f(\beta) = \frac{\beta^2}{27} = \frac{1}{3}$, $\Rightarrow \beta^2 = 9$, $\Rightarrow \beta = \pm 3$ for the interval $0 \le x \le \beta$, $\beta \ne -3$, $\Rightarrow \beta = 3$	**B1** –equating **A1** –conclusion
	$\int_{all\ x} f(x)\,dx = 1$, $\Rightarrow \int_0^3 \frac{x^2}{27}dx + \int_3^\alpha \frac{1}{3}dx = 1$ $\left[\frac{1}{81}x^3\right]_0^3 + \left[\frac{1}{3}x\right]_3^\alpha = 1$, $\Rightarrow \left(\frac{1}{3} - 0\right) + \left(\frac{\alpha}{3} - 1\right) = 1$ $-\frac{2}{3} + \frac{\alpha}{3} = 1$, $\Rightarrow \frac{\alpha}{3} = \frac{5}{3}$, $\Rightarrow \alpha = 5$ $f(x) = \begin{cases} \frac{x^2}{27} & ; \quad 0 \le x \le 3, \\ \frac{1}{3} & ; \quad 3 \le x \le 5, \\ 0 & ; \quad elsewhere. \end{cases}$ $f(0) = 0$, $f(3) = \frac{1}{3}$, $f(5) = \frac{1}{3}$	**M1** –integration & evaluation **A1** –output
		**B1** -1st part of graph **B1** -2nd part of graph
	(b). (i). Let M be the median. $\int_0^M f(x)\,dx = 0.5$, $\Rightarrow \int_0^3 \frac{x^2}{27}dx + \int_3^M \frac{1}{3}dx = 0.5$ $\frac{1}{3} + \left[\frac{1}{3}x\right]_3^M = 0.5$, $\Rightarrow \frac{1}{3} + \frac{M}{3} - 1 = 0.5$ $\frac{M}{3} = \frac{7}{6}$, $\Rightarrow M = \frac{7}{6} \times 3 = 3.5$	**B1** –identify the range **B1** –integrating **A1** –median
	(ii). Mean	

$$E(X)=\int_{all\,x} xf(x)\,dx=\int_0^3 \frac{x^3}{27}dx+\int_3^5 \frac{x}{3}dx=\left[\frac{x^4}{108}\right]_0^3+\left[\frac{x^2}{6}\right]_3^5$$

$$=\left(\frac{3}{4}-0\right)+\left(\frac{25}{6}-\frac{3}{2}\right)=\frac{3}{4}+\frac{8}{3}=\frac{41}{12}\approx 3.4167$$

M1 – integrating
M1 – evaluating
A1 –output

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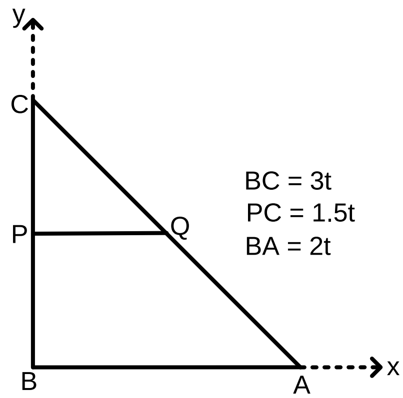


$$\text{Area } ABC=\frac{1}{2}\times 2t\times 3t=3t^2$$ **B1**

By similarity,

$$A.S.F=(L.S.F)^2$$

$$\frac{\text{Area } CPQ}{3t^2}=0.5^2$$

$$\text{Area } CPQ=0.25\times 3t^2=0.75t^2$$

$$\text{Area } ABPQ=3t^2-0.75t^2=2.25t^2$$ **B1** –areas

also, $L.S.F=\frac{\overline{PQ}}{2t}=0.5,\quad \Rightarrow \overline{PQ}=t$

Let ρ be the weight per unit area.

		Weight	Coordinates of the centre of gravity	
Figure	Area	w_i	x_i	y_i
CPQ	$0.75t^2$	$0.75t^2\rho$	$\frac{1}{3}\times 2t=\frac{2}{3}t$	$\frac{1}{3}\times 1.5t=0.5t$
ABPQ	$2.25t^2$	$2.25t^2\rho$	$\bar{x}$	$\bar{y}$
Whole lamina	$3t^2$	$3t^2\rho$	$\frac{1}{3}\times t=\frac{1}{3}t$	$\frac{1}{3}\times 3t=t$

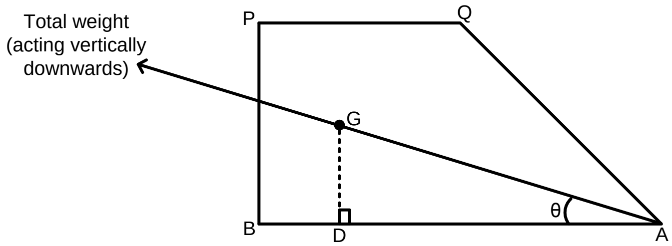
B1
B1
B1

By taking moments about x and y axes,

$$0.75t^2\rho\begin{pmatrix}\frac{2}{3}t\\0.5t\end{pmatrix}+2.25t^2\rho\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=3t^2\rho\begin{pmatrix}\frac{1}{3}t\\t\end{pmatrix}$$ **M1**

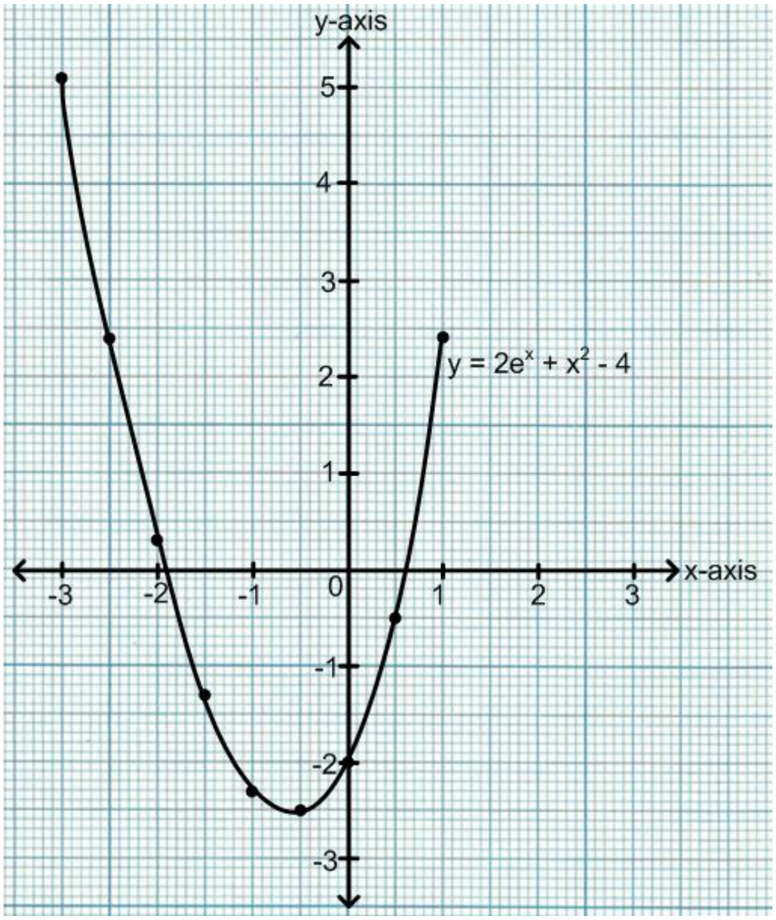
$$\begin{pmatrix}0.5t\\0.375t\end{pmatrix}+2.25\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}t\\3t\end{pmatrix}$$

$$2.25\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}t\\3t\end{pmatrix}-\begin{pmatrix}0.5t\\0.375t\end{pmatrix}=\begin{pmatrix}0.5t\\2.625t\end{pmatrix}$$ **M1**

	$$\begin{pmatrix}\overline{x}\\ \overline{y}\end{pmatrix} = \frac{1}{2.25}\begin{pmatrix}0.5t\\ 2.625t\end{pmatrix}$$ $$\overline{x} = \frac{0.5t}{2.25} = \frac{2t}{9}, \qquad \overline{y} = \frac{2.625t}{2.25} = \frac{7t}{6}$$ The distance of the centre of gravity of the lamina from AB is $\frac{7t}{6}$ and from BC is $\frac{2t}{9}$. (b). Let θ be the angle between AB and the vertical.  The coordinates of the centre of gravity, G, is given by: $$G(\overline{x}, \overline{y}) = \left(\frac{2t}{9}, \frac{7t}{6}\right), \qquad \Longrightarrow \overline{BD} = \frac{2t}{9}, \qquad \overline{DG} = \frac{7t}{6}$$ $$\Longrightarrow \overline{AD} = \overline{AB} - \overline{BD} = 2t - \frac{2t}{9} = \frac{16t}{9}$$ $$\tan\theta = \frac{\overline{DG}}{\overline{AD}} = \frac{7t}{6} \div \frac{16t}{9} = \frac{21}{32}, \qquad \Longrightarrow \theta = 33.27°$$	**A1** **B1** **B1** –for AD **M1 A1**
14	(a). $\mu = -5, \sigma = \sqrt{9} = 3$ $$P(X > 0) = P\left(Z > \frac{0-(-5)}{3}\right) = P(Z > 1.667)$$ $$= 0.5 - \phi(1.667) = 0.5 - 0.4522 = 0.0478$$ (b). $$P(\lvert X\rvert < 3) = P(-3 < X < 3) = P\left(\frac{-3-(-5)}{3} < Z < \frac{3-(-5)}{3}\right)$$ $$= P(0.667 < Z < 2.667) = \phi(2.667) - \phi(0.667)$$ $$= 0.4961 - 0.2477 = 0.2484$$ Number of scales with error magnitude of less than 3 is $$= 0.2484 \times 75 = 18.63 \approx 19 \text{ scales}$$ (c). $$n = 10, \qquad p = 0.0478, \qquad q = 1 - 0.0478 = 0.9522$$ $$\text{let,} \qquad Y \sim B(10, 0.0478)$$ $$P(Y = 4) = \binom{10}{4} \times 0.0478^4 \times 0.9522^6 = 0.00082$$	**B1** – identifying X>0 **B1** – standardizing **M1 A1** **B1** – standardising **M1** **B1** **M1 A1** **B1** – identifying it is a binomial **M1 A1**
15	(a). $$\text{let,} \qquad f(x) = 2e^x + x^2 - 4$$	

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1
$f(x)$	5.1	2.4	0.3	-1.3	-2.3	-2.5	-2	-0.5	2.4

B1



B1 –axes
B1 –all points
B1 –smooth curve

From the graph, the negative root is -1.9.

B1 –approx. root

(b).

$$f(x) = 2e^x + x^2 - 4, \qquad \Longrightarrow f'(x) = 2e^x + 2x$$

B1 –for $f'(x)$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2e^{x_n} + {x_n}^2 - 4}{2e^{x_n} + 2x_n}$$

$$x_1 = -1.9 - \frac{2e^{(-1.9)} + (-1.9)^2 - 4}{2e^{(-1.9)} + 2(-1.9)} = -1.9260$$

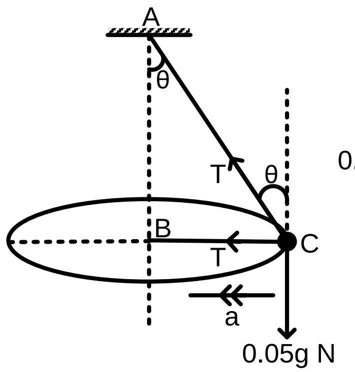
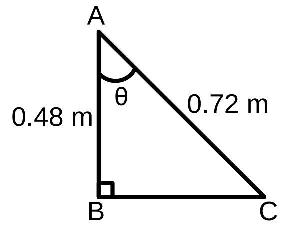
M1 B1 – substitution & output
B1 –testing for conrgence

$$x_2 = -1.9260 - \frac{2e^{(-1.9260)} + (-1.9260)^2 - 4}{2e^{(-1.9260)} + 2(-1.9260)} = -1.9257$$

M1 B1 – substitution & output

The negative root of the equation is $= -1.93\ (3\ s.f)$.

A1 –answer to 3sf

16	(a).   	
	$\cos\theta = \dfrac{0.48}{0.72}$, $\quad \Longrightarrow \theta = 48.19°$ $r = BC = 0.72\sin 48.19° = 0.54\text{ m}$	**B1** –for $\cos\theta = \frac{0.48}{0.72}$ **B1** –for finding θ **B1** –for finding radius **B1** –for converting the mass
	Resolving vertically gives, $T\cos\theta = 0.05g$ $T\cos 48.19° = 0.05 \times 9.8$ $T = 0.735\text{ N}$	**B1** –for $T\cos 48.19°$ **B1** –for $0.05g$ **M1** –for equating **A1** -output
	(b). Resolving horizontally gives, $T + T\sin\theta = mr\omega^2$ $0.735 + 0.735\sin 48.19° = 0.05 \times 0.54\omega^2$ $\omega = \sqrt{47.513} = 6.89\text{ rad s}^{-1}$	**M1** –for L.H.S **M1** –for R.H.S **M1** –for equating **A1** -output

*****END*****